

Interaction of Low - Energy Induced Gravity with Quantized Matter – II. Temperature effects

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December 1995

Abstract: *At the very early Universe the matter fields are described by the GUT models in curved space-time. At high energies these fields are asymptotically free and conformally coupled to external metric. The only possible quantum effect is the appearance of the conformal anomaly, which leads to the propagation of the new degree of freedom - conformal factor. Simultaneously with the expansion of the Universe, the scale of energies decreases and the propagating conformal factor starts to interact with the Higgs field due to the violation of conformal invariance in the matter fields sector. In a previous paper [1] we have shown that this interaction can lead to special physical effects like the renormalization group flow, which ends in some fixed point. Furthermore in the vicinity of this fixed point there occur the first order phase transitions. In the present paper we consider the same theory of conformal factor coupled to Higgs field and incorporate the temperature effects. We reduce the complicated higher-derivative operator to several ones of the standard second-derivative form and calculate an exact effective potential with temperature on the anti de Sitter (AdS) background. The physical analysis of the effective potential is performed in the framework of the high-temperature expansion.*

PACS number(s): 04.62+v

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1 Introduction

The increasing interest to the quantum field theory in an external classical gravitational field is based on the following reasons. First of all such an approach is usually regarded as a first step towards the more complete theory of quantum gravity. On the other hand, while the issue of quantum gravity itself is related to the physics at Planck scale of energies, the effects of an external gravitational background can be relevant at the energies lower than the Planck ones and therefore it can be described in more familiar theoretical models [2, 3, 4]. In particular, renormalization and the notion of renormalization group can be introduced for arbitrary curved background and this enables one to study asymptotical behaviour, phase transitions, vacuum structure and other aspects of quantum matter fields. For instance, if we are dealing with the GUT scale, then all the interactions are supposed to be asymptotically free. Furthermore, for some GUT models there is a specific phenomena of asymptotic conformal invariance [5, 4] and as a consequence at high energies we meet the quantum matter fields free of all the interactions except the conformal one with an external metric. The requirement of conformal invariance mainly concerns the scalar (Higgs) field, which fixes the value of the nonminimal parameter of scalar - curvature interaction to the conformal value $1/6$.

From the cosmological point of view, the stage when the matter fields are described by the Grand Unification Theories corresponds to inflation [6, 7]. During the inflationary epoch the Universe expands exponentially in time simultaneously with the decreasing of the character energy scale of all the interactions. Then at the lower energies the conformal invariance in the Higgs sector gets violated and as a result the scalar fields start to interact with the conformal factor of the metric. In a previous paper [1] (see also [8]) we have explored some physical consequences of this interaction. In particular it was shown that the theory of scalars interacting with the conformal factor is renormalizable and that the renormalization group flow ends in the infra-red (IR) stable fixed point with the minimally coupled scalar field free of four-scalar interactions. Furthermore, the effects of quantum conformal factor lead to a new kind of first order phase transition induced by curvature, where the Higgs field plays the role of order parameter (see also [9] where the potential of conformal factor itself was explored). Thus we have estimated the quantum effects of conformal factor to the physics of matter fields, that is in fact a new specific way of taking into account the back reaction of vacuum to the matter fields.

In all the considerations, we have used the standard perturbative rules which are equivalent to the zero - temperature approximation. At the same time for the sake of completeness one must take into account the temperature effects as well, since they could have dominated in the early Universe [10]. In the present paper we report about the study of the temperature effects in the theory of induced gravity coupled to the matter fields. Since the divergences of the theory are not affected by the temperature, we can use the renormalization group flows established in Ref. [1] for the zero-temperature case and consider the physical effects in the vicinity of the fixed points. However, the temperature effects lead to a strong modifications in the effective potential and so the physical results are essentially different in the present case.

The paper is organized as follows. In section 2 we remind the basic statements of the previous paper [1] and in particular we write down the renormalization group equations for the effective couplings. Section 3 is devoted to the calculation of the effective potential in our theory of dilaton coupled to matter in the high-temperature regime on a space-time with constant curvature. We choose the AdS background because it enables one to formulate the equilibrium state and consider the temperature effects in a consistent way [11, 12] (see also Ref [13] and references cited therein). We overcome the difficulties related with the higher derivatives which occur in the action of conformal factor by the use of some simple transformations which are especially useful for the calculation of the effective potential. As a result we obtain the general exact expression for the effective potential which depends on a lot of arbitrary parameters

including temperature, curvature, point of normalization for the Higgs field and scaling constant. Thus the general expression is not suitable for analysis and we are enforced to make some simplifications. In order to do this in section 4 we consider the special high temperature case and find that in this limit, the most of the above mentioned arbitrariness disappear.

2 Induced gravity and its interaction with matter

Let us start with some asymptotic free and asymptotically conformal invariant GUT in curved space-time. The multiplicative renormalizability of the GUT model in curved space-time requires the nonminimal terms of the form $\xi R\phi^2$ to be included into the action of every Higgs field ϕ , as well as the vacuum terms. When the radiational corrections are taken into account, the parameter of nonminimal coupling obeys the corresponding renormalization - group equations¹. As it was discovered in [5] (see also [4]), some asymptotic free models are also asymptotically conformal invariant. This means that the nonminimal coupling ξ is arbitrary at low energies while at high energies it tends to the special conformal value $1/6$. Below we consider only this class of gauge models. Since all the masses also vanish in far ultra-violet (UV) regime, at that high scale we are effectively living with free massless fields conformally coupled to the metric background. Since the interaction between matter fields are weakened then the only quantum effect is the appearance of the trace anomaly of the energy-momentum tensor. The purpose of the present paper is to explore the back reaction of this vacuum effect to the matter fields, taking into account temperature effects, which could have been important in the Early Universe.

The anomaly trace of the energy-momentum tensor appears due to the divergences and the lack of completely invariant regularization [14] (see [15] for the complete references). In the theory under discussion the anomalous trace is the combination of the vacuum beta-functions. Since the anomaly is known it enables one to calculate, with accuracy to some conformal invariant functional, the effective action of vacuum [16, 17] (see also [4] and [1]). The effective action originally arises as a nonlocal functional, but it can be written in a local form with the help of an extra dimensionless field σ which can be named as dilaton, in analogy with the string theory or as conformal factor, according to its origin. It reads

$$W[g_{\mu\nu}, \sigma] = S_c[g_{\mu\nu}] + \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \sigma \Delta \sigma + \sigma \left[k'_1 C^2 + k'_2 \left(E - \frac{2}{3} \square R \right) \right] + k'_3 R^2 \right\}. \quad (2.1)$$

Here the conformally covariant self-adjoint operator Δ is defined by

$$\Delta = \square^2 + 2R^{\mu\nu} \nabla_\mu \nabla_\nu - \frac{2}{3} R \square + \frac{1}{3} (\nabla^\mu R) \nabla_\mu. \quad (2.2)$$

The values of $k'_{1,2,3}$ are defined by the amount of the fields of spin 0, $\frac{1}{2}$ and 1 in starting GUT theory and for our purposes they are not relevant, as well as the conformal invariant nonlocal action $S_c[g_{\mu\nu}]$.

Our main supposition [1] is that the quantum effects of induced gravity, that is of the field σ , are relevant below the scale of asymptotic conformal invariance, where the nonminimal parameter ξ runs away from the conformal value. We are interested in the cosmological applications and therefore it is natural to suppose that the transition to low energies (or long distances) corresponds to some conformal transformation in the induced gravity action (2.1) and hence the classical fields and induced gravity appear in different conformal points² [18, 8, 1]. Thus it is necessary to make a conformal transformation of the metric in (2.1) and then consider the unified theory. At the same time it is much more convenient to make the conformal transformation of

¹We consider the case of one real scalar field for simplicity and therefore we deal with the single nonminimal parameter. In general, the situation may be more complicated.

²Very recently the supersymmetric generalization of Ref. [18] has been performed in Ref. [19].

metric and matter fields in the action of the last. Such a transformation corresponds to some change of variables in the path integral for the unified theory.

The only source of conformal noninvariance in the action of 0, $\frac{1}{2}$ and 1 spin fields is the nonminimal term in the scalar sector. In the framework of asymptotically conformal invariant models the value of ξ is not equal to $\frac{1}{6}$ at low energies and hence the interaction of conformal factor with scalar field arises. Introducing the scale parameter α we obtain the following action for the conformal factor coupled with the scalar field, that is

$$S = W[g_{\mu\nu}, \sigma] + \int d^4x \sqrt{-g} \left\{ \frac{1}{2}(1 - 6\xi)\phi^2 \left(\alpha^2 g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + \alpha \square \sigma \right) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} \xi R \phi^2 - \frac{1}{24} f \phi^4 \right\}, \quad (2.3)$$

where $W[g_{\mu\nu}, \sigma]$ is defined iby Eq. (2.1). Thus the interaction between scalar field and conformal factor arises as a result of the conformal transformation of the metric $g_{\mu\nu} \rightarrow g'_{\mu\nu} = g_{\mu\nu} \exp(2\alpha\sigma)$ and the matter field $\Phi \rightarrow \Phi' = \Phi \exp(d_\Phi \alpha\sigma)$, where d_Φ is the conformal weight of the field Φ . The only kind of fields which takes part in such an interaction is the scalar one, where the interaction with conformal factor appears as a result of nonconformal coupling at low energies. Hence the contributions of other matter fields to the effective potential are not important, since they can only change the values of $k'_{1,2,3}$ in Eq. (2.1).

We must take into account the quantum effects of conformal factor coupled to ordinary scalar fields and to estimate the quantum corrections to the effective potential of the last. For the sake of quantum calculations we shall follow [1] and use the background field method. We separate the fields into background σ, ϕ and quantum τ, η ones, by means

$$\sigma \rightarrow \sigma' = \sigma + \tau, \quad \phi \rightarrow \phi' = \phi + \eta. \quad (2.4)$$

The one-loop contribution to the effective action is defined as (see Sec. 3)

$$\Gamma^{(1)} = \frac{1}{2} \text{Tr} \ln \hat{H} = \frac{1}{2} \ln \text{Det} \hat{H}, \quad (2.5)$$

where \hat{H} is the bilinear (with respect to quantum fields τ, η) form of the classical action (2.3):

$$\hat{H} = \begin{pmatrix} H_{\tau\tau} & H_{\tau\eta} \\ H_{\eta\tau} & H_{\eta\eta} \end{pmatrix}, \quad (2.6)$$

where

$$\begin{aligned} H_{\tau\tau} &= \square^2 + 2R^{\mu\nu} \nabla_\mu \nabla_\nu - \frac{2}{3} R \square + \frac{1}{3} (\nabla^\mu R) \nabla_\mu + (6\xi - 1) \left[\alpha^2 \phi^2 \square + \alpha^2 (\nabla^\mu \phi^2) \nabla_\mu \right], \\ H_{\tau\eta} &= -(6\xi - 1) \left[\alpha \phi \square + 2\alpha (\nabla^\mu \phi) \nabla_\mu + \alpha (\square \phi) - 2\alpha^2 \phi (\nabla^\mu \sigma) \nabla_\mu - 2\alpha^2 (\nabla_\mu (\phi \nabla^\mu \sigma)) \right], \\ H_{\eta\tau} &= -(6\xi - 1) \left[\alpha \phi \square + 2\alpha^2 \phi (\nabla^\mu \sigma) \nabla_\mu \right], \\ H_{\eta\eta} &= -\square + \xi R - \frac{1}{2} f \phi^2 - (6\xi - 1) \left[\alpha (\square \sigma) + \alpha^2 (\nabla^\mu \sigma) (\nabla_\mu \sigma) \right]. \end{aligned} \quad (2.7)$$

The one-loop divergences show that the theory (2.3) is (at least at one-loop) renormalizable³ and therefore one can use the renormalization group method for its study. At this stage it is

³To construct the completely renormalizable theory one probably needs to use a more general metric-dilaton action introduced in [20].

more convenient to deal with an arbitrary background metric $g_{\mu\nu}$ and thus we use the approach described in [4]. The general solution of the renormalization group equations for effective action

$$\left\{ \mu \frac{d}{d\mu} + \beta_f \frac{d}{df} + \beta_\xi \frac{d}{d\xi} - \gamma_\phi \frac{\delta}{\delta\phi} - \gamma_\sigma \frac{\delta}{\delta\sigma} \right\} \Gamma[\phi, \sigma, f, \xi, g_{\mu\nu}, \mu] = 0 \quad (2.8)$$

has the form

$$\Gamma[\phi, \sigma, f, \xi, g_{\mu\nu} e^{2t}, \mu] = \Gamma[\phi(t), \sigma(t), f(t), \xi(t), g_{\mu\nu}, \mu], \quad (2.9)$$

where μ is the dimensional parameter of renormalization. Effective coupling constants obey the equations [1]

$$(4\pi)^2 \frac{df(t)}{dt'} = \beta_f = 3f^2 + 12f\alpha^2\zeta^2 + 12\alpha^4\zeta^2(\zeta - 1)^2, \quad f(0) = f, \\ (4\pi)^2 \frac{d\zeta(t)}{dt'} = \beta_\zeta = \zeta[f + 2\alpha^2\zeta(\zeta - 1)], \quad \xi(0) = \xi, \quad (2.10)$$

while γ -functions are equal to zero. Here we use $\zeta(t) = 1 - 6\xi(t)$ instead of ξ for compactness.

The study of asymptotics of the effective couplings $f(t), \zeta(t)$ can be done with the fixed point method. It is easy to see that both β functions (2.10) vanish in the physically relevant points $f = 0, \zeta = 0$ and $f = 0, \zeta = 1$. There are also three more solutions with negative f , but we do not discuss them here. The first solution corresponds to the conformal fixed point while the second one corresponds to the minimal fixed point with $\xi = 0$. The analysis of stability of the minimal fixed point has been done in the framework of standard Lyapunov's method and it was found to be stable in the IR limit $t \rightarrow -\infty$. Moreover in this limit $f \sim \xi^6 \rightarrow 0$ and therefore $f \ll \xi$. We remark that such a "far" IR regime for the gravitational effects under discussion is in fact an UV regime for the matter fields interactions and hence we can consider only the effects related with the conformal factor as relevant. It should not be so if f is the same order as ζ . The conformal fixed points turns out to be unstable [1] and it plays the role of initial point for the renormalization group flow.

The renormalization group equations (2.9) and (2.10) enable one to restore the effective action of scalar field ϕ with any preassigned accuracy [4]. However since we need to estimate the temperature corrections, which are not caused by divergences, it is much more useful to use the ζ -regularization technique choosing the special AdS background. As we shall see in the next section, in this way we shall reduce the case of higher derivative operator (2.6) to the case of second order operators for which known results are at disposal.

3 Finite temperature one-loop effective potential in AdS space.

Here we briefly recall how is defined the finite temperature one-loop effective potential within the path integral approach to quantum field theory. In a static gravitational background, the finite temperature theory is obtained by the compactification, in the imaginary time $\tau = ix^0$, of the Euclidean section of the manifold. In this way $0 < \tau < \beta$ ($\beta = 1/T$ being the inverse temperature) and the fields have to satisfy periodic boundary conditions, that is $\phi(\tau, \mathbf{x}) = \phi(\tau + \beta, \mathbf{x})$.

Let us consider the simple case of ordinary scalar field ϕ on the background of an external metric. A functional Taylor expansion of the action around the classical solution $\hat{\phi}$ gives

$$S[\phi, g] = S_c[\hat{\phi}, g] + \left. \frac{\delta^2 S[\phi, g]}{\delta\phi^2} \right|_{\phi=\hat{\phi}} \frac{\eta^2}{2} + \text{higher order terms in } \eta, \quad (3.1)$$

$S_c[\hat{\phi}, g]$ being the classical action and so the one-loop approximated theory is determined by the partition function

$$Z[\hat{\phi}, g] \sim \exp(-S_c[\hat{\phi}, g]) \int d[\eta] \exp\left(-\frac{1}{2} \int \eta A \eta d^4x\right). \quad (3.2)$$

Since we have performed a Wick rotation of the time axis in the complex plane, the metric $g_{\mu\nu}$ is Euclidean and the small disturbance operator A selfadjoint and non-negative. With these assumptions, the partition function can be formally computed in terms of the real eigenvalues of the operator A [21] and for the one-loop quantum corrections to the classical action one obtains

$$\Gamma^{(1)}[\hat{\phi}, g] = \frac{1}{2} \ln \text{Det} \frac{A}{\mu^2}, \quad (3.3)$$

where μ is an arbitrary mass, which is necessary to adjust dimensions. It will be determined by renormalization. The latter equation is valid also for a multiplet of matter fields. In such a case A is a matrix of differential operators and one gets Eq. (2.5).

The effective action can be written in the form (see for example Ref. [22])

$$\Gamma[\hat{\phi}, g] = \int \left[V(\hat{\phi}, g) + \frac{1}{2} Z(\hat{\phi}, g) g^{ij} \partial_i \hat{\phi} \partial_j \hat{\phi} + \dots \right] \sqrt{g} d^4x. \quad (3.4)$$

The latter equation implicitly defines also the contributions to the one-loop effective potential $V^{(1)}(\hat{\phi}, g)$. Using ζ -function for the definition of the determinant in Eq. (3.3), we finally get

$$V^{(1)}(\hat{\phi}, g) = -\frac{1}{2} \zeta' \left(0; x|A/\mu^2\right) \Big|_{\hat{\phi}=\text{const}}, \quad (3.5)$$

where $\zeta(s; x|A)$ is the ζ -function density related to the operator A .

In our case, the small disturbance operator (\hat{H}) is quite complicated, but it notably simplifies in a maximally symmetric background. For this reason we choose the anti de Sitter space-time, where the equilibrium state at any temperature is well defined. Now, we briefly resume some known results in AdS, referring the reader to the literature for more details (see for example [23, 11, 12, 24, 25, 13]).

Let us start by considering a scalar field in AdS satisfying the Klein-Gordon equation

$$\left(-\square + M^2 - \frac{9}{4}\Omega^2\right)\phi = 0, \quad (3.6)$$

where $M \geq 3\Omega/2$ is a constant and Ω^2 is proportional to the scalar curvature. We have

$$R_{\mu\nu} = -3\Omega^2 g_{\mu\nu}, \quad R = -12\Omega^2. \quad (3.7)$$

For this case, the zero temperature contribution $V_0^{(1)}$ to one-loop effective potential can be computed by observing that the Euclidean section of AdS is the hyperbolic manifold H^4 and on such a space the ζ -function is explicitly known [24, 26]. So one has

$$\begin{aligned} V_0^{(1)}(M) = & -\frac{1}{64\pi^2} \left[M^4 \left(\frac{3}{2} - \ln \frac{M^2}{\mu^2} \right) - \frac{M^2 \Omega^2}{2} \left(1 - \ln \frac{M^2}{\mu^2} \right) \right] \\ & - \frac{\Omega^4}{8\pi^2} \int_0^\infty \frac{x(x^2 + 1/4)}{e^{2\pi x} + 1} \ln \left(\frac{M^2 + \Omega^2 x^2}{\mu^2} \right) dx. \end{aligned} \quad (3.8)$$

The arbitrary mass scale μ has to be fixed by renormalization [27, 28]. The small curvature expansion of the latter expression reads

$$\begin{aligned} V_0^{(1)}(M) \sim & -\frac{1}{64\pi^2} \left[M^4 \left(\frac{3}{2} - \ln \frac{M^2}{\mu^2} \right) - \frac{M^2}{2} \left(1 - \ln \frac{M^2}{\mu^2} \right) \Omega^2 \right. \\ & \left. + \frac{17}{240} \ln \frac{M^2}{\mu^2} \Omega^4 \right] + O(\Omega^4). \end{aligned} \quad (3.9)$$

The finite temperature contribution $V_T^{(1)}$ has also been already computed directly using the thermodynamical formulae. It can be written in the form [13]

$$V_T^{(1)}(M) = -\frac{\Omega^4}{i\pi^3} \int_{c-i\infty}^{c+i\infty} \Gamma(1-s) \zeta_R(s) \chi(s-1; \omega_0) \left(\frac{\Omega}{T}\right)^{-s} ds, \quad (3.10)$$

where $c > 4$, $\omega_0 = M/\Omega + 3/2$ and

$$\begin{aligned} \chi(s; \omega_0) = & \frac{1}{4} \left[\zeta_H(s-2, \omega_0) - 2 \left(\omega_0 - \frac{3}{2} \right) \zeta_H(s-1, \omega_0) \right] \\ & + (\omega_0 - 1)(\omega_0 - 2) \zeta_H(s, \omega_0) \end{aligned} \quad (3.11)$$

$\zeta_R(s)$ and $\zeta_H(s, \omega_0)$ being the Riemann and Hurwitz ζ -functions respectively.

This representation of the effective potential is very useful in the computation of high temperature expansion, which we need. In fact, we can compute the integral by the residues method, by observing that the integral function has simple poles at the points $s = 4, 3, 2, 0, -1-2, \dots$ and a double pole at $s = 1$. In this way we obtain an asymptotic expansion for the potential, since the contribution of the contour integral at the (left) infinity, which we disregard, is exponentially vanishing in T . One easily obtains [13]

$$\begin{aligned} V_T^{(1)}(M) \sim & - \left\{ \frac{\pi^2}{90} T^4 - \frac{\zeta_R(3)M}{\pi} T^3 + \frac{1}{12} \left(M^2 - \frac{\Omega^2}{4} \right) T^2 \right. \\ & \left. + \frac{2}{\pi} \left[\chi(0; \omega_0) \ln \frac{T}{\Omega} + \chi'(0; \omega_0) \right] \Omega^3 T + \frac{\chi(-1; \omega_0)}{\pi} \Omega^4 \right\} + O\left(\frac{1}{T}\right). \end{aligned} \quad (3.12)$$

In our case the action assumes the form (2.3) and the one-loop effective action is defined according to (2.5), with the bilinear form (2.7). The expressions (2.7) can be notably simplified, because we are interested in the effective potential for the scalar ϕ (this means that ϕ and σ are constant) in AdS (that is $R = -12\Omega^2 = \text{const}$ and $R_{\mu\nu} = Rg_{\mu\nu}/4$). Then, the operator we are dealing with assumes the form

$$\hat{H} = \begin{pmatrix} \square^2 + U\square & x\square \\ x\square & -\square + D \end{pmatrix}, \quad (3.13)$$

where

$$U = (6\xi - 1)\alpha\phi + 2\Omega^2, \quad x = (1 - 6\xi)\alpha\phi, \quad D = -12\xi\Omega^2 - \frac{f}{2}\phi^2 \quad (3.14)$$

can be considered as some numbers since nor ϕ neither Ω do not depend on the spacetime variables. Consequently, any element of the matrix (3.13) commutes with each other and the finite temperature one-loop effective action can be obtained without special calculations taking into account the standard results written in the previous section.

To see this we observe that

$$\begin{aligned} 2\Gamma^{(1)} &= \ln \text{Det } \hat{H} = \ln \text{Det} \begin{pmatrix} \square & 0 \\ 0 & -1 \end{pmatrix} + \ln \text{Det} \begin{pmatrix} \square + U & x \\ -x\square & \square - D \end{pmatrix} \\ &= \ln \text{Det} (-\square) + \ln \text{Det} (\square - D) + \ln \text{Det} \left(\square + U + x^2\square(\square - D)^{-1}\square \right). \end{aligned} \quad (3.15)$$

Since all the operators commute the last expression can be reduced to

$$2\Gamma = \ln \text{Det} (-\square) + \ln \text{Det} (z_1 - \square) + \ln \text{Det} (z_2 - \square), \quad (3.16)$$

where

$$z_{1,2} = -\frac{1}{2}(U - D + x^2) \pm \sqrt{\frac{1}{4}(U - D + x^2)^2 + UD} \quad (3.17)$$

are the roots of the equation

$$z^2 + (U - D + x^2)z - UD = 0. \quad (3.18)$$

Now, using the above results for the effective potential V_{eff} we immediately get

$$V_{eff} = V_{cl} + V_0^{(1)} + V_T^{(1)} \quad (3.19)$$

where

$$V_{cl} = -\xi R\phi^2/2 + f\phi^4/24 \quad (3.20)$$

is the classical potential and

$$V_0^{(1)} = V_0^{(1)}(M_0) + V_0^{(1)}(M_1) + V_0^{(1)}(M_2), \quad (3.21)$$

$$V_T^{(1)} = V_T^{(1)}(M_0) + V_T^{(1)}(M_1) + V_T^{(1)}(M_2), \quad (3.22)$$

the expressions for $V_0^{(1)}(M)$ and $V_T^{(1)}(M)$ being given by Eqs. (3.8) and (3.10) respectively, with

$$M_0^2 = 9\Omega^2/4, \quad M_1^2 = z_1 + 9\Omega^2/4, \quad M_2^2 = z_2 + 9\Omega^2/4. \quad (3.23)$$

The above expression gives an exact one-loop effective potential with temperature on the AdS background for our model of induced gravity coupled to matter fields. Below we discuss some physical applications of this result.

4 First order phase transitions of the system

First order phase transitions of the system can happen for some critical values of the parameters, say ϕ_c, T_c, R_c , which are determined by the equations

$$\begin{aligned} V_{eff}(\phi_c, T_c, R_c) - V_{eff}(0, T_c, R_c) &= 0, \\ V'_{eff}(\phi_c, T_c, R_c) &= 0, \quad V''_{eff}(\phi_c, T_c, R_c) > 0. \end{aligned} \quad (4.1)$$

Higher loop contributions are significant in the determination of the exact critical parameters [29] then, for consistency, in the determination of the critical values, we have to use the approximated expressions, Eqs. (3.9) and (3.12), in the computation of the effective potential.

Let us now consider the form of the effective potential in the vicinity of the IR stable fixed point $f = 0, \xi = 0$. As it was already mentioned, the effective couplings tend to these values in a different way. In particular, $f \sim \xi^6 \rightarrow 0$. In practice, in the vicinity of the fixed point we shall keep only the lower powers of the small parameter ξ and therefore omit f almost everywhere. Indeed one must be careful in expanding the nonlinear functions like the square roots in the series of small parameter like ξ , since for big values of the scalar field, the first terms of the expansion can behave different as compared with original function. Next, we shall suppose that the system is in the high temperature phase and that $T \gg \Omega$. Using the RG based restrictions $f \sim \xi^6 \rightarrow 0$ into the zero-temperature effective potential, one obtains the renormalized expression [1], which satisfies the same normalization conditions (we keep them in what follows).

$$\begin{aligned} V_{cl} + V_0 &= 6\xi\Omega^2\phi^2 + \frac{3}{16\pi^2} (1 - 6\xi)^2\alpha^2\Omega^2\phi^2 \left[\ln \frac{\phi^2}{\mu^2} - 3 \right] \\ &\quad + \frac{9}{16\pi^2} \xi^2(1 - 6\xi)^2\alpha^4\phi^4 \left[\ln \frac{\phi^2}{\mu^2} - \frac{25}{6} \right]. \end{aligned} \quad (4.2)$$

With this restrictions, the quantities in Eq. (3.23) simplify to

$$M_0^2 = \frac{9}{4} \Omega^2, \quad M_1^2 = \Omega^2 \left(\frac{9}{4} - 12\xi \right) + (1 - 6\xi) 6\xi \alpha^2 \phi^2, \quad M_2^2 = \frac{1}{4} \Omega^2, \quad (4.3)$$

and, according to Eq. (3.22), the temperature dependent part of the effective potential becomes

$$V_T^{(1)} = \frac{3\zeta_R(3)\Omega T^3}{2\pi} \left[1 + \frac{8\xi(1-6\xi)\alpha^2\phi^2}{3\Omega^2} \right]^{\frac{1}{2}} - \frac{T^2}{2} \xi(1-6\xi)\alpha^2\phi^2 + O(\xi^{5/2}) + \text{const}, \quad (4.4)$$

where the last term is independent on scalar field and can be omitted.

In order to explore the phase transition conditions (4.1), it is convenient to introduce the dimensionless quantities

$$\chi^2 = \frac{\phi^2}{\mu^2}, \quad \hat{T} = \frac{T}{\hat{\mu}}, \quad \hat{\Omega} = \frac{\Omega}{\hat{\mu}}, \quad (4.5)$$

where $\hat{\mu}^2 = \xi(1-6\xi)\alpha^2\mu^2$ and see the one loop effective potential as a function of χ . In the approximation that we are using it reads

$$\begin{aligned} \hat{V}_{eff}(\chi, \hat{T}, \hat{\Omega}) &= \frac{V_{eff}(\chi) - V_{eff}(0)}{\hat{\mu}^4} \\ &\sim \frac{9}{16\pi^2} \chi^4 \left[\ln \chi^2 - \frac{25}{6} \right] - \frac{3\zeta_R(3)\hat{\Omega}\hat{T}^3}{2\pi} \left[1 - \left(1 + \frac{8\chi^2}{3\hat{\Omega}^2} \right)^{\frac{1}{2}} \right] - \frac{\hat{T}^2\chi^2}{2}. \end{aligned} \quad (4.6)$$

Remarkably the arbitrariness related with α drops from the potential.

The general Eqs. (4.1) are quite complicated and that is why we start the physical analysis with the particular case of the flat background metric. For this case, the effective potential

$$\hat{V}_{eff}(\chi, \hat{T}, 0) = \frac{9}{16\pi^2} \chi^4 \left[\ln \chi^2 - \frac{25}{6} \right] - \frac{\hat{T}^2\chi^2}{2} \quad (4.7)$$

has qualitatively the same form as the temperature dependent effective potential in an ordinary field theory like scalar QED [29]. The temperature contributions lead to the appearance of the positive mass term which can provide the phase transition. In our case, however, since the original theory was massless, the vacuum corresponds to broken symmetry. The second condition (4.1) gives the following relations for χ_c, T_c and the value of potential in the points of minima.

$$\begin{aligned} \frac{9}{4\pi^2} \chi_c^2 \left[\ln \chi_c^2 - \frac{11}{3} \right] &= \hat{T}_c, \\ \hat{V}_{eff}(\chi_c, \hat{T}_c, 0) &= -\frac{1}{4} \hat{T}_c^2 \chi_c^2 - \frac{9}{32\pi^2} \chi_c^4 < 0, \\ \hat{V}_{eff}''(\chi_c, \hat{T}_c, 0) &= 2\hat{T}_c^2 + \frac{9}{2\pi^2} \chi_c^2 > 0. \end{aligned} \quad (4.8)$$

The temperature dependent terms have soft behaviour at large values of ϕ and therefore the ϕ^4 terms from $V_{cl} + V_0^{(1)}$ turn out to be relevant, whereas for the ϕ^2 terms the temperature dependent part is dominating. The analysis of the potential (4.8) is straightforward. One can see that in the flat-space, both broken and unbroken phases are possible, depending on the parameter of normalization μ . The second order phase transition occurs if $\chi \gg 1$ (about two orders) that corresponds to the big value of the order parameter. In this case the magnitude of effective potential – the induced cosmological constant – does not depend on μ whereas the

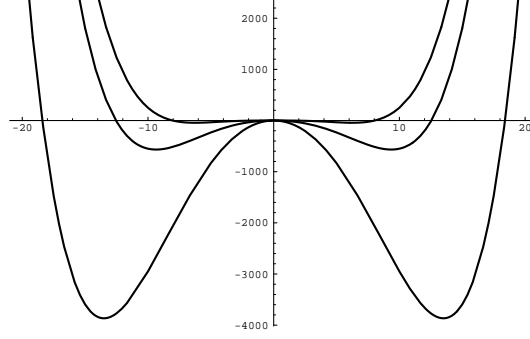


Figure 1: $\hat{V}_{eff}(\chi, \hat{T}, 0)$ for $\hat{T} = 0$, $\hat{T} = 4$ and $\hat{T} = 8$ respectively (from the top)

critical values of χ and T do. The effective potential for this case is shown in Figure 1. On the other hand, from a physical point of view it is more natural to consider μ as a character value of the order parameter χ . Then one has to put χ close to 1 and the phase is unbroken (see the first line on Figure 1).

The conditions (4.1) for the general nontrivial AdS metric are indeed much more complicated and we have analysed them numerically. It is convenient to distinguish the two regions $\hat{T} < \hat{T}_0$ and $\hat{T} > \hat{T}_0$ ($\hat{T}_0 \simeq 4.3$). In fact, in the first region $\hat{T} < \hat{T}_0$, there is always a second order phase transition for any value of $\hat{\Omega}$. The depth of the potential hole increase with Ω (remember that, to be consistent with our approximation, we have to choose $\hat{\Omega} \ll \hat{T}$). For $\hat{T} = \hat{T}_0$, a first order phase transition appears for $\hat{\Omega} = \hat{\Omega}_0 \simeq 0$ and of course, a second order phase transition for any $\Omega > \Omega_0$. In the second region, that is $\hat{T} > \hat{T}_0$, we can have both first or second order phase transitions, depending on the parameters, but the condition $\hat{\Omega} \ll \hat{T}$, notably limits the possible values of \hat{T} . In fact, one can easily verify that for $\hat{T} = 4.5$ the first order phase transition appears for $\hat{\Omega} = 0.54 \sim 0.12\hat{T}$. Then, in our approximations, we have first order phase transitions for $4.3 < \hat{T} < 4.5$ and second order phase transitions for $\hat{T} < 4.5$ (see Figure 2).

Thus one can meet three different phases depending on the value of μ . In particular, for $\chi \gg 1$ and very small values of Ω there is always broken phase but the potential in the point of minima is negative. Thus we have the second order phase transition and the result is qualitatively the same which takes place in the more simple $\Omega = 0$ case. The situation for the natural choice of normalization is also similar to the one discussed above – the phase is actually unbroken if we deal with the region $T \gg \Omega$ corresponding to our approximation. Now, if we increase the value of Ω the qualitative result becomes different but still depends on the value of the normalization constant μ . As in the previous case, the phase can be broken only if μ is about two orders less than χ . The effect of negative curvature, even if it is very small, is relevant in the vicinity of $\phi = 0$ because it changes the sign of the second derivative of the potential and therefore leads to the stability of this point.

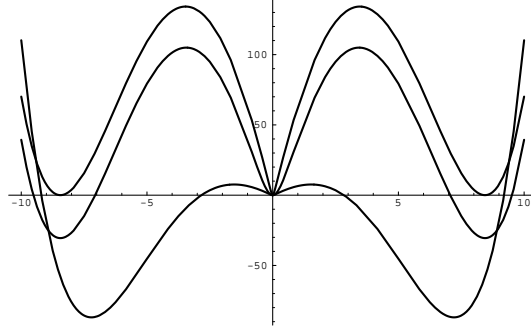


Figure 2: $\hat{V}_{eff}(\chi, \hat{T}, \hat{\Omega})$ for $(\hat{T}, \hat{\Omega}) = (4.333, 0.075)$, $(\hat{T}, \hat{\Omega}) = (4.333, 0.75)$ and $(\hat{T}, \hat{\Omega}) = (2.16, 0.075)$ respectively (from the top)

5 Conclusion

We have considered the quantum theory of conformal factor coupled to the Higgs field in the high temperature regime. It turns out that the temperature effects lead to drastical change in the effective potential of the Higgs field, as a result the quantum corrections dominate in the effective potential that was not the case in the zero-temperature approximation [1]. The temperature dependent terms have soft behaviour at large values of ϕ and therefore the ϕ^4 terms from $V_{cl} + V_0^{(1)}$ turn out to be relevant, whereas for the ϕ^2 terms the temperature dependent part is dominating. In this respect the picture is qualitatively the same as for ordinary scalar field theory in the high-temperature regime [29]. However there are crucial differences between the two theories. In the theory under consideration the quantum effects are dominating because of the specific form of the renormalization group flows (2.10) which end in the IR free point. Consequently the scalar field gets rescaled and the rescaled quantity plays a role of the order parameter. Simultaneously the classical coupling constant f vanishes in the fixed point much faster than the nonminimal parameter and the "new" scalar self-coupling is expressed in terms of the nonminimal parameter and scaling constant.

The effective potential is given by a very complicated expression and we have analysed it in the vicinity of the IR stable fixed point in the regime of high temperature and small negative curvature. The second order phase transition takes place for most of the values of the parameters of the theory. For some special values it takes place the first order phase transition too.

Acknowledgments

Authors thank I.L. Buchbinder, I.V. Tyutin and L. Vanzo for useful discussions. I.Sh. is grateful to Dipartimento di Fisica, Università di Trento for kind hospitality and INFN for the support for visiting Trento, where this work has been started. The work of I.Sh. was also supported in part by MEC-DGICYT, Spain.

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